Students and Real World Applications: Still a Challenging Mix

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Rhetoric about the importance of students being equipped to apply mathematics to relevant problems arising in their lives, individually, as citizens, and in the workplace has never been matched by serious policy or curricular support. This paper identifies and elaborates authenticity implications for addressing this issue, and describes aspects of a modelling challenge in which students were mentored to engage in problem solving located in real world settings. Characteristics of the approach and selected student responses to the challenge are provided.

The number of papers and research reports addressing the theory and/or practice of mathematical modelling with some form of connection to education continues to grow astronomically. A web-search, combining 'mathematical modelling' (both spellings) with 'education' elicited about 3,023,000 references; if model* was used in place of modelling the number increased to about 22.5 million. At the ICME-12 Congress in Korea in 2012 mathematical modelling featured as the substantive content of a Plenary Lecture, two Regular Lectures, a Topic Study Group, a Special Interest Group, and the Affiliated Study Group meetings of ICTMA[1]. Specifically within Australia, in addition to the activities of individual practitioners and researchers, the field has collectively featured in the review of research (Stillman, Brown & Galbraith, 2007), and in a special issue of the Mathematics Education Research Journal. Small wonder then that the literature contains a plethora of views concerning the theory and practice of mathematical modelling as it appears within educational settings.

That real world problem solving expertise is an espoused educational goal continues to be reinforced internationally through curriculum documents – as in the following.
From Australia: (Australian Curriculum Assessment and Reporting Authority, 2013):

> It (the national mathematics curriculum) develops the numeracy capabilities that all students need in their personal, work and civic life, and provides the fundamentals on which mathematical specialties and professional applications of mathematics are built... These capabilities enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently.

From the USA: (Common Core State Standards Initiative, 2012):

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. (p.1)

Other sources from countries such as Singapore, and regular OECD statements contain similar entries. However such abilities can only develop if mathematical experiences are drawn genuinely from these same areas of personal, vocational, and civic contexts. When the Australian curriculum statement elaborates the curricular content it includes for example, the following for the Mathematical Methods curriculum.

Purposes expressed in this way pay no more than lip service to goals of promoting student ability to apply their mathematical knowledge – they can be met at significant levels of depth, or trivially through a token interpretation of what practical problems mean.

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[1] International Community for the Teaching of Mathematical Modelling and Applications

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

The existence of similar statements in curricula from time immemorial indicate that alone they are not sufficient to produce a capability of mathematical application in the sense described above, which requires additional abilities – including to identify a feasible problem from a real context in the first place, and to decide which mathematics (from among that available to a student) is appropriate to address it. This requires a different perspective and requires different attributes, from that which looks at examples of applications within a topic area that has already been identified – useful and important though that is.

- identify contexts suitable for modelling by exponential functions and use them to solve practical problems. (ACMMM066)
- use trigonometric functions and their derivatives to solve practical problems. (ACMMM103)
- use Bernoulli random variables and associated probabilities to model data and solve practical problems. (ACMMM146)
- identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems. (ACMMM158)

And in terms of past and present performance the Programme for the International Assessment of Adult Competencies (PIAAC) which assessed adults in 25 countries in terms of proficiency in literacy, numeracy and problem-solving in a technology-rich environment has found that 8.9 million or 55 per cent of Australians achieved in the lowest two bands for numeracy. (ACER, 2013).Enough said!

This paper has two purposes:

A theoretical aim: To articulate a description of authenticity that includes dimensions necessary for assessing the validity of real world problem solving in educational settings.

A practical aim: To provide a selection of data from a modelling challenge program, that instantiates aspects of the theoretical aim.

Models of Modelling

Cyril Julie, (e.g. Julie & Mudaly, 2007) and elsewhere, uses the terms *modelling as vehicle* and *modelling as content* to distinguish between mathematical modelling used to serve other curricular needs, in contrast to its use as a means of empowering students to become independent users of their mathematics.

> Central to the debate is whether mathematical modelling should be used as a vehicle for the development of mathematics or treated as content in and of itself. A common notion associated with mathematical modelling as a vehicle is that mathematics should be represented in some context. The purpose for embedding mathematics in context is not the construction of mathematical models per se but rather the use of contexts and mathematical models as a mechanism for the learning of mathematical concepts, procedures… Mathematical modelling as content entails the construction of mathematical models of natural and social phenomena without the prescription that certain mathematical concepts, procedures or the like should be the outcome of the model-building process. (p. 504)

The tension from the other side of the fence is captured by Zbiek and Conner (2006).

> The curricular context of schooling in our country (USA) does not readily admit the opportunity to make mathematical modeling an explicit topic in the K-12 mathematics curriculum. The primary goal of including mathematical modeling activities in students' mathematics experiences within our schools typically is to provide an alternative – and supposedly engaging – setting in which students learn mathematics without the primary goal of becoming proficient modelers. We refer to the mathematics to be learned in these classrooms as "curricular mathematics" to emphasize that this mathematics is the mathematics valued in these schools … student engagement in classroom modeling activities is essential in mathematics instruction only if modeling provides our students

with significant opportunities to develop deeper and stronger understanding of curricular mathematics. (p. 89-90)

(Note that the recent US curriculum statement cited above adds a new dimension to this matter, by elevating the importance of modelling as content.)

It is not the intention here to pursue approaches to modelling that fall within the *vehicle* category. We note that many within this genre are described in the literature and here we simply identify the most common of these. Using real problem situations as a preliminary basis for abstraction (Bardini, Pierce & Stacey, 2004); Emergent modelling (Gravemeijer, 2007); word problems that use practical settings (Verschaffel et al., 2010); modelling to service other curricular needs (Zbiek & Conner, 2006). For an elaboration of these see for example, Galbraith (2011). In summary, conceptions of modelling as *vehicle*, and as *content*, rest upon different ontological premises, as well as on different epistemological descriptions. It follows that, while sharing some attributes, the associated methodologies also have distinctive differences.

At this point it is important to clarify what this paper does not set out to do. It does not contend that all mathematics should be taught in contextualised settings – modelling and applications are important abilities for students to acquire but so are other aspects of mathematics. And it does not seek to downgrade the pedagogic contributions that can be made by the skilled use of versions of *modelling as vehicle* approaches for various curricular purposes. What it does set out to do is confront the inadequacy of curricular initiatives (including the current National Statement) in providing students with opportunities to develop abilities to "solve problems arising in everyday life, society, and the workplace", despite the fact that these continue to be espoused internationally and published openly as goals of primary importance within mathematics education by the same authorities.

Central to this challenge is the construct of *Authenticity* as it applies to both choice of problems and the way in which approaches to their solution are implemented.

Authenticity

The words *authentic* and *authenticity* are favourites among those who like to promote the idea that their educational thinking really does value the goal of helping students to apply their mathematical knowledge to problems located in life outside the classroom. One source of confusion is that the term *authenticity* is used too globally, without sufficient regard to its scope and implications. Here *authenticity* is described in terms of four dimensions, derived from the attributes of real world problem solving as it is conducted by professionals, for example as argued in Galbraith (2012), and noting the requirement that this integrity be preserved in educational settings.

1. Content authenticity 2. Process authenticity
3. Situation authenticity 4. Product authenticity

Content authenticity has two aspects. Firstly the problem itself needs to satisfy realistic criteria (involve genuine real world connections), and secondly the individuals addressing it need to possess mathematical knowledge sufficient to support a viable solution attempt. Real world connections broadly encompass two types of problem viz: specific problems and life like problems where the context is real, but there is freedom in choosing the precise problem to be addressed – for example issues around deforestation.

Process authenticity refers to the approach to, and the conduct of a modelling process that results in solutions or endpoints that are defensible and robust in terms of the outcomes sought. Kaiser, Blomhoj and Sriraman (2006) remind us that while variations exist "the important thing is the commonly accepted idea about a general (cyclic) mathematical modelling process." Such processes are based on the approaches used and described by professional modellers. Figure 1 was derived from descriptions of the modelling process by Pedley (2005), an applied mathematician, and is similar to other diagrams designed over the years. Such diagrams describe the modelling process, but also act as a scaffolding aid for individuals or groups as they develop modelling skills through successive applications.

Understand the real problem situation.
Frame an appropriate mathematical question
Formulate a model, using simplifying assumptions etc
Analyse the model
Compare mathematical outcomes with reality
Modify and repeat until an adequate solution has been found.

Figure 1. Modelling Process (after Pedley, 2005).

The arrows on the left indicate a progression through stages that characterises all real world modelling projects. Progress however is almost always non-linear and the arrows on the right indicate that iterative back tracking may occur, and usually does, between any of the stages of the modelling cycle when a need is identified.

Situation authenticity is a critical dimension, as it brings conditions necessary for a valid modelling exercise into direct contact with the workplace, classroom, or other environment within which the modelling enterprise is conducted. Implications of situation can be inferred from comments (e.g., Sfard, 2008) who claimed that, the minute an *out of school problem* is treated in school it is no longer an out of school problem, and hence the search for authentic real world problems is necessarily in vain. The essential characteristic for situational authenticity is that the requirements of the modelling task drive the problem solving process. That is the nature and purpose of learner-teacher interactions, decisions about the use of technology, when and whether to work collaboratively or individually, and whether activity needs to be in a classroom, a computer lab, or some other place are determined by the problem requirements at different stages of its determination, solution and reporting. What Sfard has done is to make her conception of what it means to be *in school*, or *out of school* the definitive construct, so privileging a particular conception of what school mathematics is about, and what mathematics teaching and classrooms are allowed to be - then requiring her concept of modelling to fit the stereotype, and hence have its integrity compromised. By contrast, properly conducted modelling can challenge norms, assumptions, and stereotypes - mathematical, situational, and pedagogical.

Product authenticity while perhaps superficially obvious is both an important and an elusive concept. It is elusive, because it is not always obvious when an appropriate *product* has been achieved. Outside the classroom, when is a problem solution good enough to warrant the effort expended given that money, and/or time has run out? In the classroom, while money may not be a tangible constraint, time most certainly is. Hence assessing product authenticity involves asking how well the endpoint achieved by the modelling informs the question asked. This is a de facto way of asking whether the modelling process has been applied appropriately, and so assessing product authenticity involves looking at mathematical outcomes in two ways. Firstly to check that there are no obvious mathematical anomalies which have not been addressed, and secondly to verify that mathematical outcomes have been appropriately absorbed into implications for the real world problem being addressed. In educational settings, there can be a tendency for some students to fall in love with their models, or in a less extreme reaction, to view their constructed mathematical model as 'the model', whose right to life does not need to be subjected to inconvenient evaluations. The modelling process depicted in Figure 1 has an important part to play in quality control during this part of the endeavour.

Study Context and Data Collection

The context for the Study was a year 10/11 group of twenty students in the annual modelling challenge (2012) sponsored by A B Paterson College, Queensland. Parallel observations and data have been collected in two similar groups for the past three years. Cross group data remain to be analysed - this report in providing a snapshot of the whole, has elements of a small case study conducted over two days of intensive modelling activity. The students were drawn from a mix of schools located in south-east Queensland and Singapore, with the four members of respective groups assigned from different source schools. With the exception of one student from the host school, the program as reported by the students was unlike anything they were used to in their previous experience. During the first session (2 hours) the students were introduced to the modelling cycle through a problem involving the operation of passing lanes, as motivated by information on the duplication of a section of the Bruce Highway described on a Department of Main Roads website. All phases of the modelling cycle and their purposes were covered. Some broad suggestions for possible problem areas were provided, but the task of identifying the context and the specific problem(s) on which to work was the students' own. They had until lunchtime on the second day to complete their modelling and construct a poster describing their work, with an oral group presentation taking place in the final post-lunch session. The detail of the poster provided most of the structural data about the substantive modelling, but additionally the students completed open questionnaire items about their approach to aspects of the task – progressively as they reached different stages of the activity. Figure 2 includes the problems devised by the students, and a summary (with samples) of the main response categories for the first six questionnaire items (Q1 – Q6). The Appendix contains a summary of the task undertaken by one group of students with associated mentor comments.

Outcomes

The following observations, pertinent to the stated aims, are anchored in data that are illustrated representatively in Figure 2. Students are capable of identifying real world contexts, and articulating suitable questions amenable to their mathematical resources, with choices driven by their own interests and judgments (P, Q1, Q2, and Appendix). Questions of missing or redundant data do not even arise – the researching of issues involving real data makes the acceptance and resolution of either a natural part of the enterprise.

Problems selected by students (P)

- How many years will it take for all the trees in the forests to be cut down? What should the rate of deforestation be in order to increase the number of existing trees in 2112 by 10 or more?
- Find out the probability of someone getting cancer - focusing on lung, breast, and prostate cancers.
- When would all the ice in the North and South poles melt completely?
- A bomb threat has been discovered at the Q 1 building at 2 am. Calculate the duration required to evacuate all residents in the building.
- What course of action can the United States take in order to escape the effects of future economic downturns?

Q1 Why did you decide on this particular problem to model?
A current and important world issue (10); Interesting (4); Data availability (3)

_ Global warming has been increasing due to human activity, and the first thing that comes to mind is rising global temperatures leading to melting ice caps.
_ Deforestation has resulted in major consequences such as contributing to greenhouse gases and causing air pollution… it is imperative to know when we will run out of trees.

Q2.How did you decide on the mathematical question(s) to ask?
Relevance to issue (6); Logic (6); Group discussion (4)

_ By researching and finding statistics, then formulating a question.
_ Through discussion with group members and asking their opinions.

Q3.How did you come to choose the mathematical approach you adopted?
Favoured math approaches (6); researching data (5); relevance to question (3)

_ Graphs are easy to interpret and people are more inclined to see data in graphs
_ By analysing different approaches as a group and coming to a common choice

Q4.What were the most important assumptions you needed to make at the start?
Structural (20): [factors to ignore (7); factors central to model (13)];
Parameter values (4); Accuracy of data (4)

_ Rates of deforestation and reforestation remain constant.
_ Building was at maximum capacity with all residents fit and healthy

Q5. What were the key ideas you used to set up your model?
Formulation factors (11); problem specific detail (10); Global givens (3)

_ Consider current amount of trees as well as rates of reforestation and deforestation
_ General behaviour of humans and their reaction time

Q6. What were the most important pieces on mathematics used in the modelling?
Algebra- equations etc (11); Data/statistics (8): Graphs (5)

_ Graphs, statistics, equations
_ Use of algebra equation, as well as rates of change and conversion of units

Figure 2. Problems and illustrative questionnaire response data.

Students make suitable simplifying assumptions as required by their chosen approach (Q3, Q4 and Appendix). Activity is centred on the group problem solving task, and is strongly influenced by students, being essentially driven by the stage and needs of the problem solving process (Q3, Q4, Q5, Q6 and Appendix). In addition to ensuring that individual contributions and group problem solving proceed productively, a teacher's role as implied by mentor comments in the Appendix becomes crucial when the students present a model for independent evaluation. Here specific knowledge is required, but more importantly suggestions as to how the initial (almost inevitably simple) model might be refined. (Although not represented in Figure 2 teacher - student interactions were mostly initiated by the latter in the form of questions.)

Levels of each of the dimensions of authenticity can be identified in the students' modelling choices and procedures. The final outcomes in each case require critical evaluation, refinement, and indicate the need to revisit aspects of the solution process – which in the present context was precluded by the time factor.

Concluding Reflection

Fundamental distinctions (in ontologies) lie beneath respective purposes of teaching and learning conventional curricular mathematics, and learning to apply existing mathematical knowledge to solve real or life-like problems. The latter goes far beyond 'feel good' statements about the importance of being able to apply mathematical knowledge. A persistent tension lies behind the reluctance of many to undertake modelling that seems to be so different from what has conventionally become accepted as classroom mathematics. Such tensions would be eased by recognising that the extended and iterative nature of real world problem solving could be incorporated by including it as a parallel component within course structures - with different classroom norms, assessments, and methodologies.

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Appendix

Summary of Sample Problem (depletion of forests)

Rationale

When the world runs out of trees many unfortunate situations will occur. Some of the main effects would be loss of many animal species due to their loss of habitat, and many others will be endangered. Wood will become a rare and valuable resource, and from this paper and honey. Tree roots and leaves are used in some medicines that would become non existent. Fruits like apples and oranges will no longer exist, along with some types of nuts. A variety of insects will become extinct, and many organisations such as Greenpeace will riot. And from the space where trees were, weeds and plants will overgrow. Trees, being a source of renewable oxygen, help contain climate change, and without them, more greenhouse gases will cloud the atmosphere. As you can see, the loss of trees is a huge problem, and we must find a renewable source for our materials.

Question(s): How many years will it take for all the trees in the forests to be cut down? What should the rate of deforestation be in order to increase the number of existing trees in 2112 by 10% or more?

Assumptions:

A1.Rates of deforestation and reforestation are constant

A2.Trees are identical, and we use Eucalyptus trees A3.Each tree has a circular canopy

A4.Canopies of the trees do not intersect, but rather only touch each other

A5.Weather in each forest is identical A6.No natural disasters

Key Variables: X (current number of trees in world's forests); n (number of years for all world's trees to be cut down); Y (rate of reforestation: trees/yr); Z (rate of deforestation: trees/yr):

Solution and Interpretation: Equations for the model ($X – n(Z-Y) = 0$ and $X – n(Z-Y) \geq 1.1X$, with current value of X estimated from internet research and assumptions A2, A3, and A4.

A key component involved estimation of area occupied by a typical tree from researched value of average canopy length. This enabled international data given in terms of forest areas to be converted to numbers of trees as required by the model. Estimated year of final depletion was 2582 with consistent interpretations.

Evaluation: In their evaluation the students revisited their assumptions and basically stood by their approach as a robust estimate. However in suggesting extensions to their project, they indicated that major forests needed to be treated individually in determining rates of deforestation, deforestation and current number of trees. This effectively means that A2, A3, and A4 would be revamped in terms of the relevant localities.

Report: The project was described, explained, and illustrated via a poster display and verbal presentation.

Comment by Mentor: The students demonstrated a sound grasp of the modelling process that had been introduced to them through a prototypical example. They researched their topic well, and identified assumptions that enabled them to develop an initial model that gave insight into both orders of magnitude, and what needed to be done to address the problem. Relevant data were assembled from internet research and used to estimate the values of essential variables. They were not critical enough of the robustness of their solution in terms of future impacts that might be expected – for example national efforts to reduce deforestation, and increase rates of reforestation, as the resource diminishes. Their own suggested extension would however, following further work, provide for interesting geographical comparisons using trees typical of different regions. This would mitigate their stated optimism that the choice of a eucalyptus tree was sufficiently typical to investigate the problem on a global scale. In an ongoing educational context their approach would support refinements of their model by exploring (for example with spreadsheets) the impact of varying rates of reforestation and deforestation as functions of time, or in terms of the magnitude of forests remaining.